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Combined effects of Hall current and variable Viscosity on Non-Newtonian MHD flow past a stretching vertical plate

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Abstract:

This paper investigate the effects of Hall currents on free-convective steady laminar boundary-layer flow, past a semi-infinite vertical plate, for large temperature differences. A uniform magnetic field is applied perpendicular to the plate. The fluid thermal conductivity is assumed to vary as a linear function of temperature. The fluid viscosity is assumed to vary as a reciprocal of a linear function of temperature. The usual Boussinesq approximation is neglected. The nonlinear boundary layer equations governing the problem under consideration are solved numerically by applying an efficient numerical technique based on the shooting method. The effects of the magnetic parameter *M*, the density / temperature parameter *n*, the thermal conductivity parameter *S*, the viscosity temperature θ_r are examined on the velocity and temperature distribution as well as the coefficient of heat flux and shearing stress at the plate.

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Nomenclature

C_f	coefficient of skin friction	U	velocity component in x-direction
c_p	specific heat at constant pressure	υ	velocity component in y-direction
e_b	blackbody emissive power	W	velocity component in z-direction
$e_{b\lambda}$	Planck' function	**Greek Symbols**	
f	dimensionless stream function	α	thermal diffusivity
g	acceleration due to gravity	β	coefficient of thermal expansion
Gr	Grashof number	η	pseudo similar variable
k	thermal conductivity	λ	wavelength
K_λ	absorption coefficient	μ	Dynamical viscosity
K_R	Rosseland absorption coefficient	ν	kinematical viscosity
L	characteristic length	θ	dimensionless temperature
N	radiation parameter	θ_r	viscosity/temperature parameter
Nu	Nusselt number	ρ	Density
τ	shearing stress	σ	Stefan-Boltzmann constant
ξ	dimensionless streamwise coordinate	X	streamwise coordinate
ψ	stream function	**Subscripts**	
n	density temperature parameter	W	property at the wall
S	thermal conductivity parameter	∞	freestream condition
T	Temperature	**Superscripts**	
u	velocity component in x-direction	$'$	differentiation with respect to η only

Introduction:

Free- convective flows driven by temperature differences are of great interest in number of industrial applications .Buoyancy is also of importance in an environment where differences between land and air temperature can give rise to complicated flow patterns, and in enclosures such as ventilated and treated rooms and reactor configurations. Pohlhausen [1] studied this problem, for the first time, using the momentum-integral method. A similarity solution for this problem was solved, firstly by Ostrich [2]. Since then, various effects, such as the effect of magnetic field on the free- convective flow past a vertical surface, which plays an important role in several engineering applications, have been studied.

Most of the problems concerning free-convection were solved assuming moderate temperatures. However, from a practical point of view, many modern industries, especially in nuclear engineering and space technology, depend strongly on very high temperatures and accordingly, requires considering radiative heat transfer as well as convective heat transfer. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. In an ionized gas where the density is low/or the magnetic field is very strong, the conductivity will be tensor. The conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both electric and magnetic fields. This phenomenon is called Hall effect, in this case it is become more convenient to take Hall current effect into accounts, and so relevant works have been published (see, for instance, Abo-Eldahab [3], Abo-Eldahab [4] and Popand Watanabe [5]. Abo-Eldahab [6] analyzed the problem of mixed convection heat transfer near an inclined isothermal stretching sheet in the presence of blowing/suction, internal heat generation/absorption and transverse magnetic field. In the present work we consider the MHD mixed convection heat transfer in the presence of a strong extended magnetic field, so Hall currents are included in the formulation.

Recently, Aboeldahab [7] studied the radiation effect on heat transfer in an electrically conducting fluid at a stretching surface with a uniform –free stream.

Previous studies of convective flow along vertical plates in the presence of radiation were restricted, in general, to the case where the temperature difference between the plate and the fluid was small. In this case, the fluid's physical properties such as its viscosity and thermal conductivity may be taken as constant .Also, for small temperature differences, the Boussinesq approximation can be used to treat the fluid density as a constant in the continuity equation, energy equation, and convective terms in the momentum equation and treat it as a variable only in the buoyancy term of the momentum equation .

In situations where there is large temperature differences between the plate and the fluid, especially when radiative heat transfer takes place, the fluid's physical properties are affected by the high temperature and they can no longer be regarded as constant .Also, in this case, the Boussinesq approximation can no longer be used.

Some recent studies for radiating fluids have taken into account variations of the physical properties with temperature. For example, Aboeldahab [8] studied radiation and variable density effects on the free convective flow of a gas past a semi-infinite vertical plate and showed that for high-temperature differences the Boussinesq approximation leads to substantial errors in velocity and temperature distributions. Aboledahab and El Gendy[9] studied the radiation effect on convective heat transfer in an electrically conducting fluid at a stretching surface with variable viscosity and uniform free stream .They showed that the flow characteristics are markedly affected by the variation of viscosity with temperature . Aboeldahab and Salem [10] studied the radiation effect on the MHD free – convective flow of a gas past a semi-infinite vertical plate with variable viscosity. Also, they showed that the flow characteristics are markedly affected by the variation in viscosity with temperature. Recently, Jaber [11] studied the effect of chemical reaction, Ion and Hall currents over a moving cylinder.

Hence, in the present work, we study the combined effects of Hall currents and variable viscosity on the MHD free-convective flow of an optically thin gray gas past a semi-infinite vertical plate with variable density, viscosity and thermal conductivity for high temperature differences neglecting the Boussinesq approximation . The nonlinear boundary layer equations, governing the problem, are solved numerically by applying an efficient numerical technique based on the shooting method. The velocity and temperature distributions as well as the coefficient of heat flux and the shearing stress at the plate are determined for different values of the thermal conductivity parameter S, the viscosity-temperature parameter θ_r, the magnetic field M, and the radiation parameter N

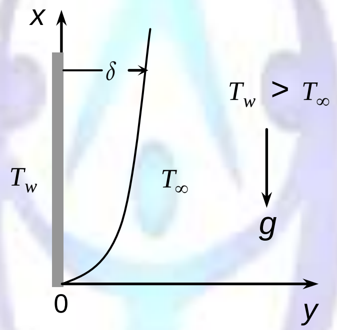

Figure 1 Physical coordinate system

2- Mathematical formulation

A steady laminar free-convective flow of a viscous gray gas in the optically thin limit past an isothermal semi-infinite vertical plate is considered. The x-axis is chosen along the plate and the y –axis is taken as normal to it (see Fig.1).

A uniform magnetic field is applied transversely to the direction of the flow. The magnetic Reynolds number is taken to be small enough so that the induced magnetic field can be neglected.

The viscous dissipation ; the radiative heat flux in the x – direction, in comparison to the y-direction ; and the velocity of the gas far away from the plate are assumed to be negligible .

The fluid thermal conductivity is assumed to vary as a linear function of temperature in the form (see [16])

$$K = k_\infty \left[1 + b(T - T_\infty)\right] \qquad (1)$$

Where b is a constant depending on the nature of the fluid . In general , $b > 0$ for fluids such as water and air , while $b < 0$ for fluids such as lubricating oils.

The fluid viscosity is assumed to vary as a reciprocal of a linear function of temperature in the form (see Lai and Kulacki, ref. 17)

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \gamma(T - T_\infty)\right] \qquad (2)$$

or

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$$\frac{1}{\mu} = a[T - T_r] \tag{3}$$

Where

a= $\frac{\gamma}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\gamma}$

Where T_r and a are constants and their values depend on the reference state and the thermal property of the fluid γ . In general a > 0 for liquids and a < 0 for gases.

Then the steady laminar two-dimensional free-convective flow in the presence of radiation is governed by the following boundary- layer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + g\rho_\infty\beta(T - T_\infty) - \frac{\sigma_o B_o}{(1+m^2)}(u + mw) \tag{5}$$

$$\rho\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right) + \frac{\sigma_o B_o}{(1+m^2)}(mu - w) \tag{6}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) \tag{7}$$

The physical problem suggests the following initial and boundary condition

$$\text{u} = \text{v} = \text{w} = 0,\ T = T_\omega \quad \text{at y}= 0 \ ;$$

$$\text{u} \longrightarrow 0,\ \text{w} \longrightarrow 0,\ T \longrightarrow T_\infty \quad \text{as y} \longrightarrow \infty \tag{8}$$

Introducing the following dimensionless variables

$$\psi = X^{\frac{3}{4}} f(\xi,\eta)\ ,\quad \xi = X^{\frac{1}{2}} L^{\frac{-1}{2}},\quad \eta = X^{\frac{-1}{4}} y,\quad \theta = \frac{T - T_\infty}{T_w - T_\infty},\quad w = g(\xi,\eta)\sqrt{L} \tag{9}$$

The continuity equation is satisfied by

$$u = \frac{\partial \psi}{\partial y}\ ,\quad v = -\frac{\partial \psi}{\partial x} \tag{10}$$

From (9) and (10) we find that

$$u = X^{\frac{1}{2}} f^{/}\ ,\quad v = -\frac{1}{4} X^{-\frac{1}{4}}\left(3f + 2\xi\frac{\partial f}{\partial \xi} - \eta f^{/}\right) \tag{11}$$

Using the above transformation the governing equations are reduced to:

$$\frac{1}{2} f^{/2} - \frac{3}{4} ff^{//} + \frac{1}{2}\xi[f^{/}\frac{\partial f^{/}}{\partial \xi} - f^{//}\frac{\partial f}{\partial \xi}] = \frac{1}{\theta_r - \theta}\left(\frac{-\theta^{/}}{\theta_r - \theta} f^{//} - f^{///}\right) + Gr\,\theta - \frac{M}{1+m^2}\left(\xi f^{/} - m g\right) \tag{12}$$

$$\frac{\xi}{2} f^{/}\frac{\partial g}{\partial \xi} - \frac{1}{4} g^{/}\left(3f + 2\xi\frac{\partial f}{\partial \xi}\right) = \frac{-\theta^{/}}{(\theta_r - \theta)^2} g^{/} - \frac{1}{(\theta_r - \theta)} + \frac{\xi M}{1+m^2}\left(\xi m f^{/} - g\right) \tag{13}$$

$$\frac{1}{2}\xi[f^{/}\frac{\partial\theta}{\partial\xi}-\theta^{/}\frac{\partial f}{\partial\xi}]-\frac{3}{4}f\theta^{/}=N\left(\theta^{/^{2}}+\theta\theta^{//}-\frac{1}{s}\theta^{//}\right) \quad (14)$$

The boundary conditions are transformed into

$$\eta=0 \qquad : f^{\backslash}=o \qquad , \theta=1 \qquad , 3f+2\xi\frac{\partial f}{\partial\xi}=0$$

$$\eta\longrightarrow\infty \qquad : f^{\backslash}\longrightarrow o, \qquad \theta\longrightarrow 0 \quad (15)$$

where $s=n(T_w-T_\infty)$, $\theta_r=\dfrac{T_r-T_\infty}{T_w-T_\infty}$

$$N=\frac{K_\infty s}{\rho C_p} \qquad , G_r=g\beta(T_w-T_\infty) \ , \quad M=\frac{\sigma_o B_o\sqrt{L}}{\rho_\infty}$$

And primes denote differentiation with respect to η only

The most important characteristics of the flow are shearing stress at the plate

$$C_{fx}=-\frac{2\mu\sqrt{l\xi}}{\rho u_\infty}f^{//}(\xi,0)$$

$$C_{fz}=-\frac{2\mu\sqrt{l\xi}}{\rho u_\infty}g^{/}(\xi,0).$$

And the rate of heat transfer at the plate (Nusselt number)

$$N_u=K\sqrt{l}\,\xi\theta^{/}(\xi,0).$$

3- Results and Discussion

Equation (12) - (14) with the boundary conditions (15) , are approximated by a system of nonlinear ordinary differential equations replacing the derivatives with respect to ξ By two – point backward finite differences with step – size $h = 0.1$ this system is solved numerically by using the fourth-order Runge-Kutta method algorithm with a systematic estimation of $f^{//}(\xi,\eta)$, $g^{//}(\xi,\eta)$ and $\theta^{/}(\xi,\eta)$ by the shooting technique to obtain $f(\xi,\eta)$, $g(\xi,\eta)$ and $\theta(\xi,\eta)$.

The value of η at infinity is fixed at 2; the requirement that the variation of velocity and temperature distribution is less that 10^{-9} between any two successive iteration is employed as the criterion of convergence. We use the symbolic computational software Mathematica to solve this system.

In view of Equations (12)-(14) density can be written in the form

$$\rho=\frac{1}{a(T_w-T_\infty)}$$

Figure 1 and 2 show the effect of the Grashoff number Gr on the primary and secondary flow velocities within the boundary layer. In which the increasing of the Grashoff number Gr is to increase the dimensionless velocities $f^{/}$ and g. Figure 3 and 4 present typical profiles for the primary velocity $f^{/}$ and secondary velocity g for different values of magnetic parameter M. The increasing of the magnetic parameter M is to decrease the dimensionless primary velocity and to increase the dimensionless secondary velocity g. The decreasing of $f^{/}$ is due to the increasing of the Lorentz force, which opposes the flow. Form Figures 5 it is observed that the dimensionless secondary velocity g decreases as the Hall parameter m increases.

Figures 6 and 7 show as expected, that the dimensionless velocities f' and g increase as the thermal conductivity parameter S increases. This is because as S increase the thermal conductivity of the fluid increase. This increase in the fluid thermal conductivity increases the fluid temperature and accordingly the fluid velocity. Also, it is observed from Figures 8 and 9, that as the viscosity -temperature parameter θ_r increases the dimensionless velocities f' and g increase. An increase in the density temperature parameter n means an increase of the velocity in the fluid particles due to an increase in the buoyancy forces (the density variation with temperature increases). Hence as n increases the fluid will be under two forces: the first force increase the velocity of the fluid due to the increase in the buoyancy forces and the second force decrease the velocity of the fluid due to the decrease in the temperature.

Table 1 shows that the dimensionless wall-velcioty gradient $f''(\bar{x},0)$ increases as n ,S, θ_r , θ_w and N increase where as it decreases as M increases. Moreover, the dimensionless rate of heat transfer- $\dot{\theta}'(\bar{x},0)$ increases as n and θ_r increase as it decrease as S , M , θ_w , N increases.

4- Concluding Remarks

In this paper , we have studied the effects of radiation on the MHD free convective steady lamina boundary layer flow past an isothermal semi-infinite vertical plate , for high temperature differences , the fluid is considered to be electrically conducting in the sence that it is ionized due to radiation .

This paper demonstrate the fact that the Boussinesq approximation gives substantial errors in the velocity and temperature distribution for high temperature differences.Therefore, to conclude more accurate results the density variation has to be taken into consideration in the continuity equation , energy equation and all terms of the momentum equation.

Besides, it is observed that:

1) The increasing in the Hall currents yields to a decreasing in the fluid secondary velocity, the fluid temperature the dimensionless wall-velocity gradient and the rate of heat transfer from the plate to the fluid .

2) The increasing in the magnetic parameter yields to an increasing in the fluid temperature the dimensionless wall-velocity gradient and the rate of heat transfer from the plate to the fluid and a decreasing in the fluid velocity

3) The increasing in the thermal conductivity parameter yields to an increasing in the fluid velocity, the fluid temperature the dimensionless wall-velocity gradient and the plate to the fluid.

4) The increasing in the viscosity-temperature parameter yields to an increasing in the fluid velocity . The dimensionless wall-velocity gradient and a decreasing in the fluid temperature and the rate of the heated transfer form the plate to the fluid.

5) The increasing in the density -temperature parameter yields to an increasing in the fluid velocity .and the dimensionless wall-velocity gradient and a decreasing in the fluid temperature and the rate of the heated transfer form the plate to the fluid .

6) the increasing in the Grashoff number yields to an increasing in the primary and secondary flow velocities, also to a decreasing in the fluid temperature and the rate of the heated transfer form the plate to the fluid .

Table 1. Variation of dimensionless wall-velocity gradient and dimensionless rate of heat transfer at the plate with the dimensionless θ_r, n, s, Gr, M and m.

θ_r	n	s	Gr	M	m	$f''(0,\xi)$	$g'(0,\xi)$	$-\theta'(0,\xi)$
0.4	1.2	0.19	0.1	0.1	1	0.0695989	0.000289243	0.31365
0.35	1.2	0.19	0.1	0.1	1	-0.0101006	0.0000665997	0.3172
0.365	1.2	0.19	0.1	0.1	1	0.00711129	0.0000981729	0.316333
0.4	1.2	0.19	0.1	0.1	1	0.0695989	0.000289243	0.31365
0.4	6	0.19	0.1	0.1	1	0.0809877	0.000339247	0.318017
0.4	15	0.19	0.1	0.1	1	0.0829863	0.000348511	0.318735
0.4	1.2	0.01	0.1	0.1	1	0.0437672	0.000196894	0.286958
0.4	1.2	0.2	0.1	0.1	1	0.0709943	0.000295669	0.315445
0.4	1.2	0.25	0.1	0.1	1	0.0811126	0.000335957	0.324759
0.4	1.2	0.19	0.01	0.1	1	0.00334127	0.0000129676	0.318956
0.4	1.2	0.19	0.1	0.1	1	0.0695989	0.000289243	0.31365
0.4	1.2	0.19	0.35	0.1	1	0.12623	0.000592708	0.30917
0.4	1.2	0.19	0.1	0.1	1	0.0695989	0.000289243	0.31365
0.4	1.2	0.19	0.1	1.5	1	0.056674	0.00328567	0.314713
0.4	1.2	0.19	0.1	2	1	0.0532747	0.00403512	0.31497
0.4	1.2	0.19	0.1	3	1	0.0477018	0.00522331	0.315365
0.4	1.2	0.19	0.1	0.1	1	0.0695989	0.000289243	0.31365
0.4	1.2	0.19	0.1	0.1	2	0.070312	0.000234604	0.313587
0.4	1.2	0.19	0.1	0.1	3	0.070555	0.000176774	0.313567

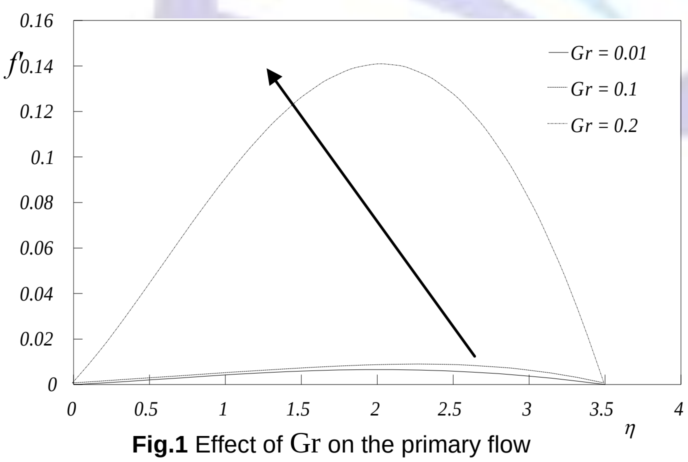

Fig.1 Effect of Gr on the primary flow velocity profiles f' with M = 0.1, n=1.2, s = 0.19, θ_r =0.4 and m = 1

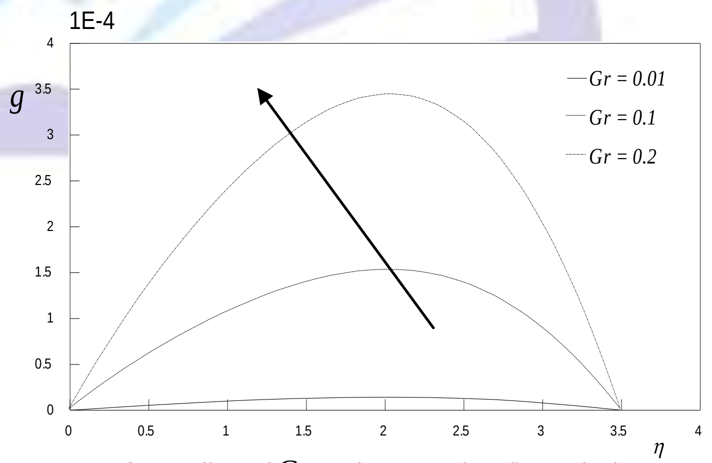

Fig.2. Effect of Gr on the secondary flow velocity profiles g with M = 0.1, n=1.2, s = 0.19, θ_r =0.4 and m = 1

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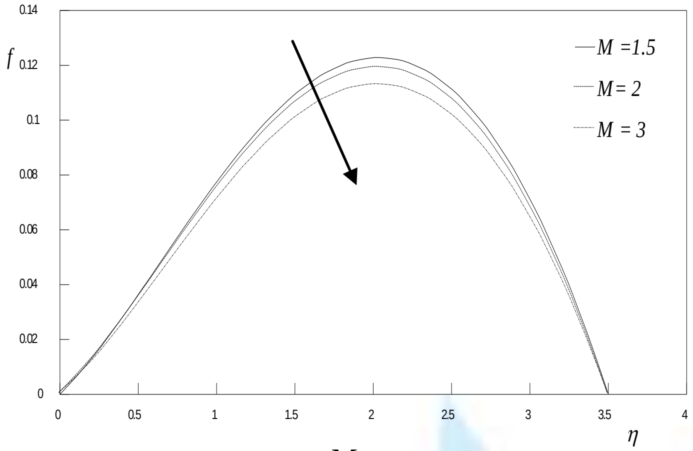

Fig.3 Effect of M on the primary flow velocity profiles f' with Gr = 0.1, n=1.2, s = 0.19, θ_r =0.4 and m = 1

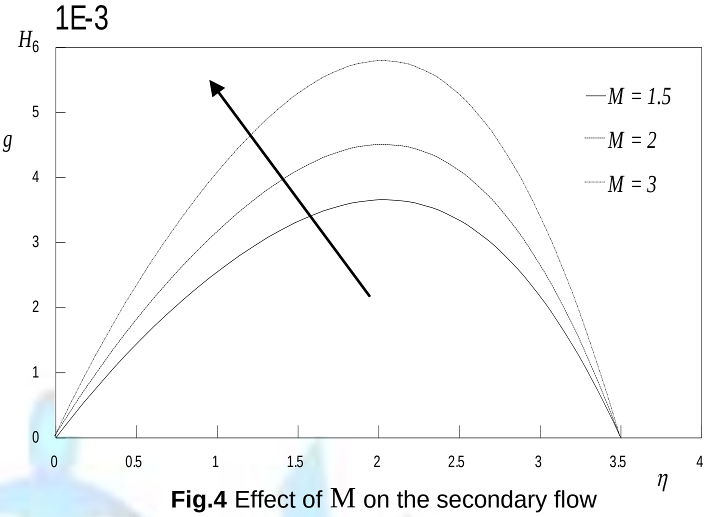

Fig.4 Effect of M on the secondary flow velocity profiles f' with M = 0.1, n=1.2, s = 0.19, θ_r =0.4 and Gr = 0.1

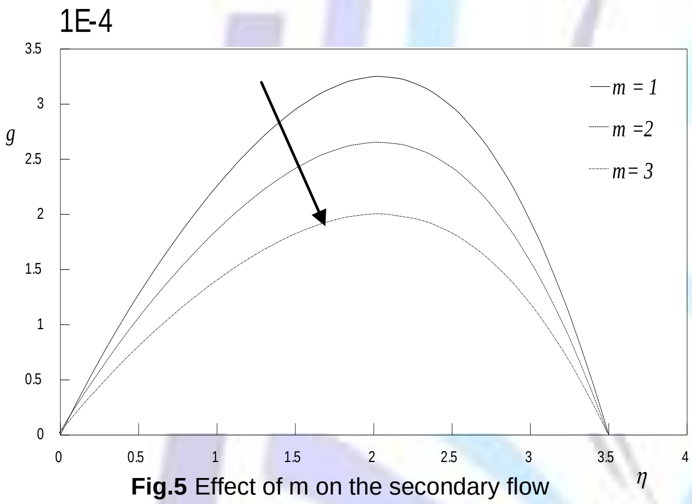

Fig.5 Effect of m on the secondary flow velocity profiles f' with M = 0.1, n=1.2, s = 0.19, θ_r =0.4 and Gr = 0.1

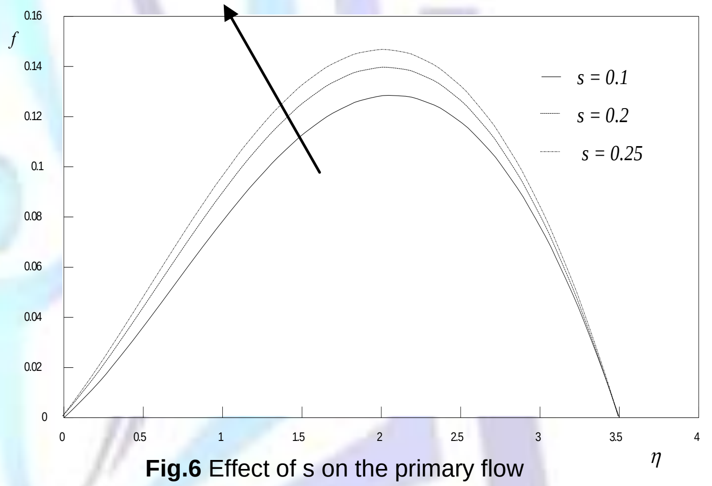

Fig.6 Effect of s on the primary flow velocity profiles f' with Gr = 0.1, n=1.2, M= 0.1, θ_r =0.4 and m = 1

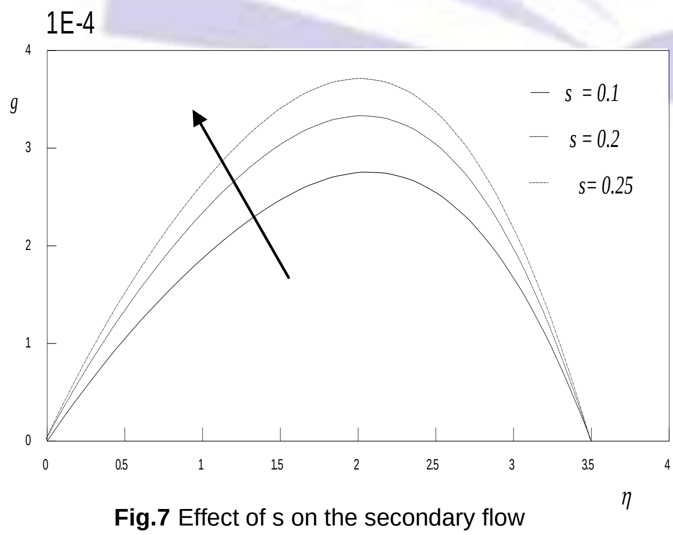

Fig.7 Effect of s on the secondary flow velocity profiles f' with M = 0.1, n=1.2, M = 0.1, θ_r =0.4 and Gr = 0.1

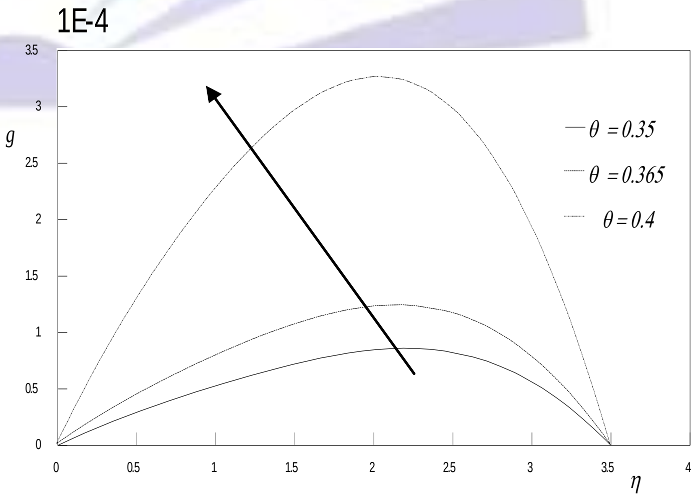

Fig.8 Effect of θ_r on the secondary flow velocity profiles f' with Gr = 0.1, n=1.2, M= 0.1. **s** = 0.19 and m = 1

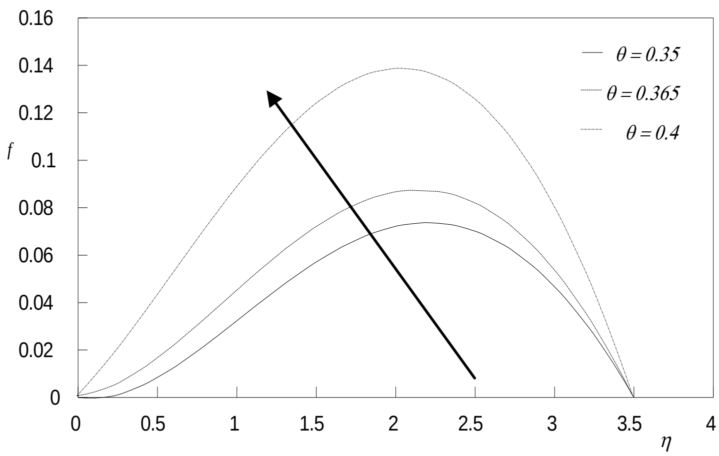

Fig.9 Effect of θ_r on the primary flow velocity profiles f' with Gr = 0.1, n=1.2, M= 0.1, **s** = 0.19 and m = 1

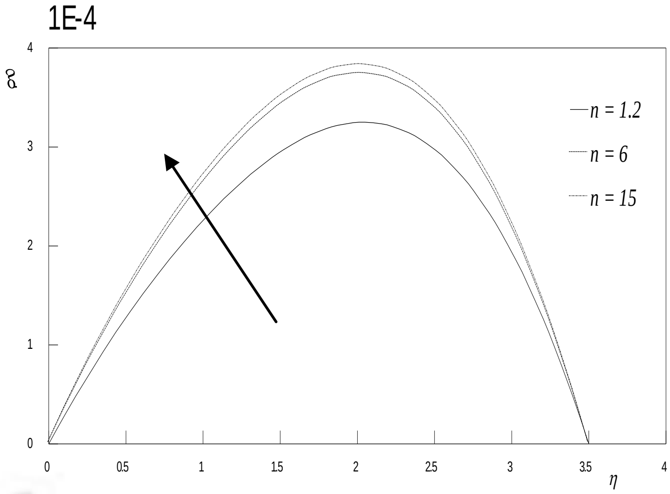

Fig.10 Effect of **n** on the secondary flow velocity profiles f' with M = 0.1, m = 1, θ_r = 0.4, **s** = 0.19 and Gr = 0.1

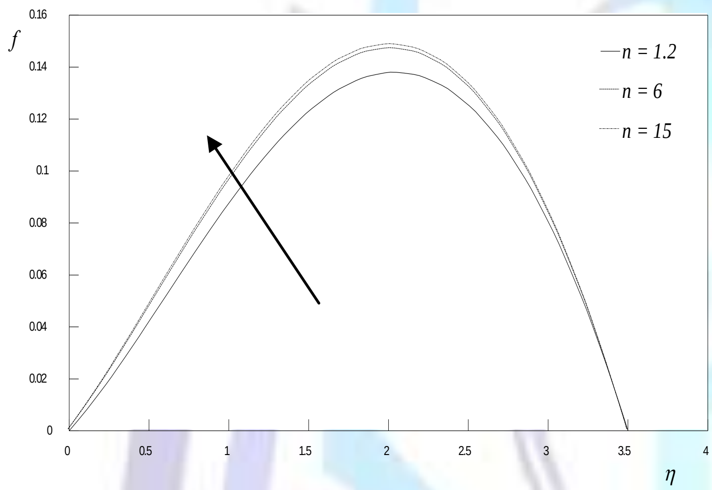

Fig.10 Effect of **n** on the primary flow velocity profiles f' with Gr = 0.1, θ_r = 0.4, M= 0.1, **s** = 0.19 and m = 1

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